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The Premium of Marine Protected Areas: A Simple Valuation Model

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Abstract *The article addresses the induced cost, the premium, from establishing a marine protected area in a deterministic model of a fishery. Outside the protected area, the fishery is managed optimally through total allowable catch quotas. The premium is found to be increasing and convex along the protection parameter. Biological measures are introduced to increase the understanding of the mechanisms in the bioeconomic system. Time-series solutions show that the net return per unit of fish increases after the protected area is established.*

Key words Bioeconomics, dynamic programming, fisheries management, marine protected areas, migration, modeling, optimization, renewable resources.

JEL Classification Codes C61, Q22, Q57.

Introduction

A no-take marine protected area (MPA) is a geographically defined area, covering parts of a fish stock's habitat that is closed to fishing (Hannesson 1998). The main issue in this article is how the formation of an MPA causes changes in the value of a fishery. In particular, we relate the size of the MPA to the value of the fishery, focusing on the change in value as the protected area size changes. The area size is treated as *exogenous* in the model. We are thus not interested in 'the optimal size' of the MPA. We are rather investigating the implications of MPAs to economic and, to a certain extent, biological measures.

As evident in the literature, the use of MPAs introduces a range of potential benefits, and the implementations are well motivated. MPAs may produce benefits in numerous ways and in many different aspects. The value of a fishery, however, is one of the few things that may suffer (*i.e.*, decrease) from the establishment of marine reserves. It is thus both appropriate and meaningful to analyze this separately, as we do here.

The model is deterministic, and we introduce perfect enforcement of the optimal harvest rate. It turns out that any degree of protection; that is, any fraction of the habitat under protection, *reduces* the equilibrium value of the fishery. Both

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Hannesson (1998) and Holland and Brazee (1996) arrive at the same conclusion, although under slightly different assumptions. This is expected, as the protection acts as a limitation in the exercise of the optimal management of the fishery. Accordingly, we are not interested in the protection decision as such. Rather, we assert that the decision to protect habitat results from some political process or is based on considerations separate from those addressed in this article. We emphasize that this is both realistic and relevant, particularly due to the poor success of traditionally managed fisheries. Further, changes in the protection policy may result from changes in the political landscape and, intriguingly, for reasons far removed from resource management issues.

Marine protected areas are by many held to produce different benefits (not assessed in this investigation) and reduce risks. It is in the risk perspective we invoke the term 'premium.' We review some of the extensive literature on these issues, MPAs in general, and clarify our perspectives. First, we assert that for protected areas to be of any interest in a socioeconomic perspective, these benefits and the value of reduced risk should be compared to the reductions in value of the fishery. The reductions are indicated in this article. As the model is deterministic, the uncertainty involved in fisheries management may well dilute our results. However, the deterministic solution provides the first-order approximation to the stochastic problem. For small levels of uncertainty, the deterministic solution and, in particular, our solution, is relevant. For high levels of uncertainty, the benefits of an MPA should equal the levels of reduction in value.

The interest in marine protected areas (or marine reserves; marine sanctuaries) has been increasing over the last 15 years, among both scholars and managers. The literature addressing different aspects of MPAs is extensive. Regarding biological issues, the literature presents many good reasons to consider MPAs as a management tool for fish stocks. Reserves can reduce the variance of a population (Conrad 1999) and catches (Hannesson 2002); raise the spawning and exploited biomass level (Gell and Roberts 2003; Guénette and Pitcher 1999); and improve recruitment (Guénette and Pitcher 1999), density, and diversity (Halpern 2003; Sanchirico 2000). Further, MPAs can protect against extinction (Flaaten and Mjølhus 2005) and management failure (Lauck *et al.* 1998). "[…] nearly any marine habitat can benefit from the implementation of a reserve" Halpern (2003, p. 117). When it comes to the economic aspects, the literature is less conclusive. Hannesson (1998) demonstrates that reserves cannot yield economic benefits in a deterministic model, and compared to pure *total allowable catch* (TAC) policies, reserves reduce efficiency (Anderson 2002). However, Sanchirico and Wilen (1998) demonstrate that under certain conditions marine reserves represent an economic benefit to the system and improve overall harvest levels. An enlightening review targeting policy implications is Grafton, Kompass, and Schneider (2005). Further, Sanchirico (2000) provides a thorough discussion of potential costs and benefits arising from MPAs and asserts that reserves can increase *nonconsumptive use values* and control fishing effort. Hastings and Botsford (1999) find equivalence in yield from reserves and traditional management, however, in the *sink-source* case. Nonetheless, there is a wide consensus in the literature that MPAs need to be combined with other management tools to provide positive results (*e.g.*, Apostolaki *et al.* 2002; Flaaten and Mjølhus 2005; Grafton, Kompass, and Schneider 2005; Sanchirico 2000; and Sumaila 2001). We agree with this view and think it is important to study the interplay between a protected area and other management regimes. In this article, we combine an MPA with a TAC policy enforced on the remaining grounds; *i.e.*, grounds open to fishing.

Uncertainty is an important issue in the literature on MPAs, and in many cases it is the main motivation behind the promotion of marine reserves. Reserves are held to reduce the risk from poor stock assessment and collapse (Arnason 2001; Lauck *et*

al. 1998; Sanchirico 2000), uncontrollability, and economic shortsightedness (Clark 1996), *inter alia*. In particular, MPAs address "persistent, *irreducible* (our emphasis) scientific uncertainty pertaining to marine ecosystems" (Lauck *et al*. 1998, p. 72). Further, terms like 'insurance value' (Grafton, Kompass, and Schneider 2005; Sumaila 2001) and 'bet hedging' (Lauck 1996; Sanchirico 2000) are frequently in use in relation to MPAs. "Bet hedging is usually thought to involve a cost, or 'premium,' in terms of a decrease in expected benefits, which is accepted in order to achieve a reduction in risk" (Lauck *et al*. 1998, pp.74–5). This is our perspective on marine protected areas. They reduce the value of the fishery, and the reduction is comprehended as a premium. As already mentioned, different types of other benefits (not directly related to reduced risk), may result from the implementation of MPAs. While we do not address any of these benefits, we do produce a numeric valuation of the premium. We implement the optimal harvesting strategy; *i.e*., maximizing the pecuniary value of the fishery, in all cases in this article.[1] Thus, we provide a lower benchmark for the premium of marine reserves. Note that even though our model does not explicitly account for uncertainty, both MPAs and the *feedback* type of harvest policy deal with uncertainty in different ways. We will discuss this in detail.

Many authors try to assess the intrinsic value or economic performance of MPAs. Hannesson (1998), Holland and Brazee (1996), Sanchirico and Wilen (1998), and Sumaila (2001) are some. Among these, our model is closely related to that of Hannesson (1998). However, Hannesson (1998) regards the equilibrium solution under open-access in the unprotected area and compares it to open-access and optimal harvesting in the entire area. Further, his diffusion process is slightly different than ours. Our model is dynamic, and we always implement the optimal strategy. Holland and Brazee (1996) present a dynamic model. The biological submodel is different from ours. Further, they discuss different effort levels in the fishery. They implement a fixed unit price of fish; we invoke a demand curve. Sanchirico and Wilen (1998) also consider different biological structures from us and, as Hannesson, invoke rent dissipation. Also, the price is fixed in their analysis. Finally, Sumaila (2001) relates game theoretic perspectives to recruitment failure. Notably, our work combines the dynamic approach with the optimal harvesting strategy. Further, the fact that the unit price of fish varies with the size of catch has implications for the value of the fishery.

The Model

The model is an aggregated and deterministic formulation in continuous time. It is also *autonomous*, which means that the involved functions are independent of time. Let us first look at the construction of the premium.

The induced cost of implementing a marine reserve is simply the result of a reduction in value of the fishery. Thus, we compare the value of a fishery where an MPA has been established to an identical fishery with no protected area. Let P denote the *premium* of the marine protected area implemented upon given initial conditions. This is formulated as:

$$P(X^0, Y^0; s) = \frac{V_0 - V(X^0, Y^0; s)}{V_0}, \tag{1}$$

[1] More specifically, we compare the value of the fishery, managed through an optimal TAC policy, with and without the MPA present. The stocks within and outside the reserve are treated as substocks, interacting through a *diffusion-type* migration term.

where X^0 and Y^0 are initial conditions on the stock levels out- and inside the protected area, respectively.[2] s is defined as the *degree of protection* and is a number between 0 and 1; it is the proportion of the total carrying capacity under protection.[3] As indicated above, s is treated as a parameter in the problem. That is, the degree of protection is not treated as a control variable; s is exogenous. The MPA divides the the stock into two substocks, one outside the reserve and one under protection. These are denoted by X and Y, respectively. Furthermore, we define $V_0 = V(X^0, Y^0; 0)$; the value of the fishery with no protected area. (We will stick to the following convention: superscripts relate to the time variable, and subscripts relate to the protection parameter.) The premium is normalized according to V_0. Hence, the premium is given as a share of the total profits accruing from the fishery with no reserve present. Please note that we are comparing steady-state values. That is, the arguments in the premium are derived from the steady state. We will use optimal control theory to optimize the value of the fishery. Before we move on to stating the optimization problem, a discussion of the stock dynamics is required.

The parameter $s \in (0,1)$ measures the degree of protection. That is, the share s of the entire habitat area is protected from fishing. The carrying capacity of the environment is spread uniformly throughout the habitat. It follows that s measures the share of the carrying capacity under protection. By uniformity we attain that the only difference between the protected and unprotected stock is the obvious difference in harvest rate. Further, the assumption of uniformity distinguishes our model from a sink-source model of the population (cf. Sanchirico and Wilen (1998)). The total carrying capacity is given by K. The carrying capacity within the reserve is hence $s \cdot K$, and outside the reserve it is $(1 - s) \cdot K$.

Since we perform numerical calculations of the premium, the model must be specific. Growth is assumed to be density dependent. The *density of fish* is simply defined as the ratio between actual biomass and carrying capacity (*i.e.*, the density outside the sanctuary is given by $X/(1-s)K$, where X now, of course, measures the biomass found outside the sanctuary). It follows that the density is 0 when the stock is extinct and 1 when the biomass equals the carrying capacity. For all practical purposes, the density is somewhere between 0 and 1. Natural growth obeys the logistic growth law *in each subarea*:

$$F(X; K) = rX\left(1 - \frac{X}{K}\right),$$

where r is the intrinsic growth rate, X is the biomass, and K is the carrying capacity. This implies that the aggregated growth will not agree with the logistic growth for

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[2] A more intuitively natural definition of the premium could be to let it only take one stock level argument, as the stock presumably is uniformly dispersed prior to the implementation of the MPA. However, our formulation is more convenient as we compare steady-state values of the premium, where the density dispersion is no longer uniform. This is explained later.

[3] It is convenient to think of protection in terms of the carrying capacity, which is what matters for the outcome of the model. For practical purposes, however, it is better to refer to the geographical proportion of the habitat under protection, as MPAs are, indeed, a geographical concept. Under assumptions of *uniformity*, the factors comprising the carrying capacity; *e.g.*, food and shelter, are uniformly spread throughout the entire habitat, and these two different interpretations of protection coincide. We want to keep both notions of MPAs in mind, as they are useful in different ways. Further, assuming uniformity of the carrying capacity rules out the possibility of a so-called sink-source system (Sanchirico and Wilen 1998).

the entire area whenever the densities in the subareas are different. There is one simple reason for this; that the logistic growth law is nonlinear in the stock. In general, one cannot add up two (or more) logistic functions to obtain a new logistic function. This results in a difference in growth for a given total biomass for the two biological models, something that is widely ignored in the literature.[4] Bischi, Lamantia, and Sbragia (2006) and Flaaten and Mjølhus (2005) recognize and discuss the issue, however. The former paper suggests an extra term in the growth function to correct for the difference; the latter investigates both ideas. Notwithstanding, we find it natural to let the growth in one area depend on local conditions and be independent of conditions in more remote areas. It makes perfect sense with the *patchy distribution* motivation, which we will return to.

We model the migration of fish between the areas according to Hannesson (1998). The same idea is considered by Conrad (1999), Flaaten and Mjølhus (2005), and Sumaila (2001), *inter alios*. The idea behind the migration model is inspired by the way the natural sciences consider how substances (typically gases) diffuse through a membrane due to differences in density, based on random movements and the theory of probability. The *net* diffusion always smoothes densities and points into the low-density area. The migration term in our model depends only on the difference in density. We write:

$$\phi\left(\frac{Y}{sK} - \frac{X}{(1-s)K}\right),$$

where ϕ is the *rate of migration* and determines how quickly the difference in density will be smoothed. The limits $\phi \to 0$ and $\phi \to \infty$ have the following interpretations. The first yields a one-dimensional problem (the protected stock is not interesting as there is no interaction) with a reduced carrying capacity ($sK \leq K$). The second yields a system with instantaneous redistribution of densities, with total carrying capacity K; *i.e.*, it is equivalent to the no protection scenario ($s = 0$). For any positive and finite rate of migration, we have a two-dimensional system; that is, two substocks interacting through migration. We always refer to the protected habitat as *one* area, but extending the model to consider several protected areas is fairly straightforward. Then, Y and sK would measure the aggregated biomass and carrying capacity for all the protected areas. Some assumptions regarding the migration rate would take care of the rest. The way we have formulated the model, ϕ can be understood to depend on the size and properties of the interface between the protected and unprotected areas. Keeping ϕ fixed (as we do) implies that the properties of the interface are basically unchanged for all s. If one thinks this construction is a bit too rigorous or not general enough, there is another way to think of the model: think of a fish stock with a *patchy distribution*, where one or more of the patches are under protection (Hannesson 2002, Sanchirico and Wilen 2002). ϕ would then contain information about the distance, current conditions, and so on between the

[4] The growth in our MPA model will depend on the distribution of fish between the different areas. Note that when $s = 0$, our model is equivalent to a one-dimensional model with instantaneous redistribution. In our model, redistribution between the areas is based on diffusion, and is not generally equivalent to instantaneous redistribution. The diffusion model is expounded in the next paragraph. We believe that the distribution does matter, and the MPA model can be comprehended as a first approximation to a spatial growth model.

patches. These quantities would, for simplicity, be assumed to be constant.[5]

We are now ready to state the dynamic equations. t denotes time:

$$\left.\begin{aligned} \frac{dX}{dt} &= F(X;(1-s)K) + \phi\left(\frac{Y}{sK} - \frac{X}{(1-s)K}\right) - H \\ \frac{dY}{dt} &= F(Y;sK) - \phi\left(\frac{Y}{sK} - \frac{X}{(1-s)K}\right) \end{aligned}\right\}. \tag{2}$$

It is readily seen that the migration term adds to the stock in the low-density area, while reducing the stock in the high-density area. Further, the migration terms cancel in aggregate; *i.e.*, the migration does not change the total biomass level. Also note that the carrying capacity is different in the two areas, according to the discussion above, and that there is no harvest term in the second equation.

Now we turn to the economic aspects of the model. There are many things that influence the economy of a fishery. We simplify and consider two issues: production and demand. The production function has the Schaefer form:

$$H = qEX,$$

where q is the *catchability coefficient*, and E is effort. As before, X is the stock biomass and H is the harvest rate. It simply tells us that a certain amount of effort applied to a certain stock level yields a certain harvest. There is a constant cost per unit effort; the costs are given by $C(X,H) = cE$. If we substitute E according to Schaefer, we obtain:

$$C(X,H) = c\,\frac{H}{qX}.$$

There is a downward sloping demand for fish; the inverse demand function has slope $-d$. The price of fish is given by $P(H) = p - dH$, where p is the maximum price the market is willing to pay. A higher price leads to full substitution. We are now able to describe the net profit function:

$$\Pi(X,H) = (p - dH)H - \frac{cH}{qX}.$$

This is the same profit function as suggested by Kugarajh, Sandal, and Berge

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[5] Actually, the first notion of the model; one big area divided into two smaller subareas, implies fundamentally different assumptions in the cases of one area ($s = 0$, $s = 1$) and of two areas [$s \in (0,1)$]. As long as $s = 0$ or $s = 1$, there is only one area, and the fish are uniformly dispersed over the entire habitat. Any change in the stock level at one place in the area, *e.g.* by harvesting activity, will, with infinite speed, spread throughout the habitat. An instant later, the density of fish is again uniform. But whenever $s \in (0,1)$, changes in stock level will instantly redistribute in each area, but changes across the areas will not occur faster than the migration term allows. This is why growth in the model does not compare to the growth in the one area model, as discussed earlier. One way to avoid this problem is to motivate the model with the notion of patchy distributions. However, this is more of a philosophical discussion, and we will not go into it further.

(2006). The interpretation of the relationship c/pq is important. It is readily seen that if $X = c/pq$, it is not possible to draw rents from the fishery. c/pq is often referred to as the *open-access solution* or the *bionomic equilibrium* (Clark 1990), which is characterized by zero profits. Further, note that profits depend on X, the biomass in the unprotected area (*i.e.*, where fishing activity is allowed), but not on Y, and that there is a *stock effect* on costs. In our model, the stock effect represents an economic protection mechanism on the stock and implies that rational fishing activity cannot eradicate the stock. Also, the downward sloping demand guarantees that the *Hamiltonian*, which will be introduced later, is non-linear in the harvest variable. Hence, *bang-bang* type solutions are avoided.

The optimization problem may now be formulated as follows:

$$V(X^0, Y^0; s) = \max_H \int_0^\infty e^{-\delta t} \Pi\big[X(t; s), H(t; s), t\big] dt, \tag{3}$$

subject to constraints on the control, growth in stock, and initial conditions:

$$\left.\begin{aligned} \frac{dX}{dt} &= F(X; (1 - s)K) + \phi\left(\frac{Y}{sK} - \frac{X}{(1 - s)K}\right) - H \\ \frac{dY}{dt} &= F(Y; sK) - \phi\left(\frac{Y}{sK} - \frac{X}{(1 - s)K}\right) \\ X(0; s) &= X^0, \qquad Y(0; s) = Y^0, \qquad H(t; s) \geq 0 \end{aligned}\right\}, \tag{4}$$

where $X(t;s)$ and $Y(t;s)$ is the biomass in the unprotected and protected area, respectively; $H(t;s)$ is the harvest rate; δ is the rate of discount; t is time; Π is a measure of profit; $F(\cdot)$ is the natural growth in biomass as a function of biomass (where the carrying capacity parameter changes with the degree of protection); K is the total carrying capacity of the environment. X^0 and Y^0 are the initial conditions of the biomasses; *i.e.,* the biomasses at time zero.[6]

To make both calculations and later discussions easier, we choose to change the scale of both variables and parameters. A consequence of this transformation is that stocks are measured in *local densities* instead of biomass. Local density is the ratio between the biomass in an area and the local carrying capacity. To avoid confusion and more notation, we use the same symbols as before, now in lower case. The main difference is that we talk about densities instead of biomass. The maximization problem in equations (3) and (4) now appears as:

$$\max_h \int_0^\infty e^{-\gamma\tau} \pi\big[x(\tau; s), h(\tau; s)\big] d\tau, \tag{5}$$

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[6] Note that if the density of fish is uniform at time zero, we have:

$$\frac{Y^0}{s} = \frac{X^0}{1 - s} \Rightarrow Y^0 = \frac{s}{1 - s} X^0.$$

Thus, assuming initial uniformity allows a definition of the premium where it only takes one argument.

subject to:

$$\left.\begin{aligned} \dot{x} &= f(x) + \frac{\omega}{1-s}(y-x) - h \\ \dot{y} &= f(y) - \frac{\omega}{s}(y-x) \\ x(0;s) &= x^0,\ y(0;s) = y^0,\ h \geq 0 \end{aligned}\right\}, \tag{6}$$

where $\dot{x} \equiv \mathrm{d}x/\mathrm{d}\tau$, x^0 and y^0 are initial conditions on the densities, and:

$$\left.\begin{aligned} \pi(x,h) &= h\left(1 - \frac{x_0}{x} - bh\right) \\ f(x) &= x(1-x) \end{aligned}\right\}, \tag{7}$$

where x and y are the local densities of fish in the protected and unprotected areas, respectively. Further, $b = (1-s)K(dr/p) \equiv (1-s)b_0$, and f is the transformed logistic function. $x_0 = (c/pq)[1/(1-s)K]$ is the open-access *density* level, $\tau = rt$ is the time variable, $\gamma = \delta/r$ is the discount rate, and $h = H/[r(1-s)K]$ is the harvest rate. Note that the migration rate, ϕ, has been replaced with $\omega = \phi/rK$ combined with a fraction depending on the degree of protection, s. That is, we need to control for the different sizes of the areas when we use densities instead of absolute biomass. We will from now on refer to ω as the normalized rate of migration. The value of the fishery is now given by (5), and we refer to it as $v(x^0, y^0; s)$. It relates to the absolute value as $V(X^0, Y^0; s) = pK \cdot v(x^0, y^0; s)$. Thus, the factor pK will cancel from P, such that:

$$P(X^0, Y^0; s) = \frac{V_0 - V(X^0, Y^0; s)}{V_0} = \frac{v_0 - v(x^0, y^0; s)}{v_0} = P(x^0, y^0; s),$$

where $v_0 = v(x^0, y^0; 0)$.

The scaling transformation is merely a technical construction, and the structure of the model is obviously unchanged. The relations and effects stand out more clearly, however.[7] It might seem unnecessary to state the model in both absolute and relative terms, but the patient reader will discover that both formulations are useful to us.

The *current value Hamiltonian* is a useful tool for these kinds of optimization problems. It is given by:

$$\mathscr{H}(x, y, h, m, n) = \pi(x,h) + m \cdot \chi(x, y, h) + n \cdot \psi(x, y), \tag{8}$$

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[7] As an example of the more lucid structure, consider the scaling of the discount rate: $\gamma = \delta/r$, where δ is the absolute discount rate and r is the intrinsic growth rate in the logistic function. The scaling tells us that what matters for the optimal solution of the problem is the ratio between interests from alternative investments and the return from the stock. This is not a particularly deep insight. It is, however, obvious from the scaling transformation.

where m and n are the *current value multipliers* (or *shadow values*, *costates*), and $\chi(x, y, h)$ and $\psi(x, s)$ are the right-hand sides of the dynamic conditions in (6), all associated with the two state variables, x and y, respectively. The first-order conditions for solutions of (5) and (6) are found in Kamien and Schwartz (1991) and are given by:

$$\left.\begin{aligned} \dot{x} &= \mathscr{H}_m \\ \dot{y} &= \mathscr{H}_n \\ \dot{m} &= \gamma m - \mathscr{H}_x \\ \dot{n} &= \gamma n - \mathscr{H}_y \\ h &= \arg\max_h (\mathscr{H}) \end{aligned}\right\}, \tag{9}$$

where subscripts denote partial derivatives. The first two equations in (9) are equivalent to the dynamic conditions in (6). Remember that the degree of protection, s, is only a parameter in this system. When we produce numerical solutions, a new s poses a new problem, which in turn requires a new optimal solution.

$\dot{x}$ and $\dot{y}$ are given in (6). For the sake of completeness, we write out the expressions for $\dot{m}$ and $\dot{n}$ according to equation (9).

$$\dot{m} = \gamma m - \frac{x_0}{x^2} h - m\left(1 - 2x - \frac{\omega}{1 - s}\right) - n\frac{\omega}{s}$$

$$\dot{n} = \gamma n - m\frac{\omega}{1 - s} - n\left(1 - 2y - \frac{\omega}{s}\right).$$

For *inner solutions*, $h = \arg\max_h (\mathscr{H})$ in equations (9) yields:

$$\mathscr{H}_h = 1 - \frac{x_0}{x} - 2bh - m = 0.$$

Otherwise, we have $h = 0$. Note that when considering the equilibrium solution; that is, putting all dot equations in (9) equal to zero, $\dot{y} = 0$ produces the inequality $y \geq x$. The inequality is strict for any practical purposes [$x, y \in (0,1)$, $0 < \omega < \infty$].

Before we move on to the results, there are a few things to be aware of with this model. The first is a technical issue; the two limits $s \to 1$ and $s \to 0$. The problem should be clear from the equations in (2); we end up with zero in the numerators. Analyzing what happens with the dynamics in these limits is an interesting discussion in itself, but we are not going into it in full detail. First, we are interested in marine protected areas, which implies $s > 0$. This makes the zero limit uninteresting. Notwithstanding, only variables and structures related to the protected area become 'singular' in the lower limit, but the area is nonexistent. Thus, our model behaves nicely in all interesting variables in the limit and is equivalent with the corresponding one-dimensional model. Thus, the one-dimensional model serves as a form of benchmark for the MPA model. Sandal and Steinshamn (1997, 2001) consider the corresponding one-dimensional model in more general terms and provide and discuss feedback solutions; the optimal control depends on the stock level. Feedback solutions are a powerful tool to deal with uncertainty. "Feedback rules […] repre-

sents adaptive management as the control variable changes immediately when new knowledge about the state variable is available" (Sandal and Steinshamn [2001, p. 420]). In the opposite situation, where we approach full protection ($s \to 1$), we expect the steady-state stock to approach the carrying capacity and harvests to drop to zero. This is not always true in our model. That said, a sanctuary covering the entire habitat is not interesting in the context of this article. Hence, we will focus on $s \in (0,1)$ and not consider the upper limit.

A weakness in the model (and in the field in general) is the lack of knowledge and information about the rate of migration. According to McGarvey and Feenstra (2002), three forms of movement models have been postulated. These are reviewed in Quinn and Deriso (1999). The diffusion model is one of them (Quinn and Deriso 1999, pp. 402–3). Fournier, Hampton, and Sibert (1998) estimate movement parameters in the diffusion sense for South Pacific albacore. Their estimates, translated into our normalized migration rate, suggest a rate in the area of 0.1 to 0.3. We investigate a larger range of migration rates. Regarding MPAs, McGarvey (2004) estimates the migration rate of fish stocks from marine sanctuaries. He is, however, not using the diffusion model, and his estimate is difficult to compare given our understanding of the parameter.

We have chosen not to incorporate uncertainty in the model. One motivation for that is that there is very much uncertainty about the uncertainty itself, the probability distribution, the functional form of it, and so on. Additionally, we would be concerned with uncertainty with regard to several aspects of the model. Adding a stochastic term, of which we have little empirical knowledge and know little of its functional form, does not necessarily improve the model. We should develop tools which are robust to imperfect information about uncertainty (which the information about uncertainty inherently always is) and, if possible, independent of it.

Nonetheless, a short discussion of the implications of uncertainty follows. Under uncertainty, the value of a fishery is, in general, changed. The question is how the change is distributed along the protection parameter. A way to look into this would be to consider single, exogenous shocks to the stock levels, either in both areas or only in the exploited areas. If the underlying uncertainty was identically distributed, it would be natural to compare positive and negative shocks in terms of the value of the fishery. As the market mechanism in our model is linear, that part of the model would be neutral to identical positive and negative shocks. The optimal response in harvesting would induce larger changes in harvest upon negative shocks, due to the general form of the optimal harvesting strategy (an example of the optimal harvest function is given in the appendix; figure A1). Moreover, it is likely that the stock would 'recover;' *i.e.*, return to a steady state faster from a positive shock, as the growth function usually is steeper to the right of an equilibrium, (because the equilibrium usually is to the right of the *maximal sustainable yield* (MSY) stock level for positive discount rates). That is, negative shocks are more persistent and have a larger impact on harvest. However, given the complexity of the model, we would not be surprised if the increase in value from positive shocks dominated the reduction in value from negative shocks for a limited range of reserve and shock sizes. This should be investigated further. However, the shocks in stock levels are probably not identically distributed. As such, one may also investigate the situation from initial stock levels different from the equilibrium levels and compare effects from identical shocks under different management regimes (*i.e.*, reserve regimes and the no reserve regime). Although this seems readily feasible, an entirely new (numerical) analysis is required to address theses issues, as it requires the optimal harvesting strategy to be recalculated with respect to uncertainty. Particularly, a realistic stochastic process must be formulated and motivated. This must be properly connected to the migration pattern and all other parts of the biological model.

Results

Deriving analytic results that are comprehensible and manageable is a difficult and often impossible task in models like this.[8] Instead, we resort to numerical investigations of the model. In the next section we briefly explain how the numerical solutions are obtained before we turn to the results. Firstly, we address the premium and consider different aspects explaining our results. Moreover, we consider different biological measures and examine some time-series solutions.

The Numerical Analysis

We use optimal control theory to produce the optimal management strategy. Since the problem is *autonomous*, we are able to produce *feedback solutions*, solutions only dependent on state variables; *i.e.*, the densities, and thus are independent of time. In particular, we use dynamic programming. The maximization problem is formulated in continuous time, with continuous state variables. We follow the discretization process in Grüne and Semmler (2004) to make state and time variables discrete and to form the *Hamilton-Jacobi-Bellman equation* for our problem. However, our approach is a slight variation to Grüne and Semmler (2004) when it comes to the maximization procedure, as we assume the control variable; *i.e.*, the harvest rate, to take values in the continuum $[0, \infty)$. (Grüne and Semmler (2004) assert that the control variable takes values in a predetermined fixed set of discrete values.) This is possible because the control variable enters in the profit function (7) to the second degree. We are thus able to identify a unique control which maximizes π. The method implies computing a fix point of a dynamic programming operator. More technical details are found in the appendix.

The parameter values listed in table 1 are based on data presented by Kugarajh, Sandal, and Berge (2006). Two relevant parameters are not present in the table: ω and s. As mentioned, little data exist on the migration parameter, ω. We will focus on a range of values, exploring the effects from various levels of migration.

The price parameter, p, and the carrying capacity, K, are only relevant in the link between the relative and absolute formulation of the premium. Appropriate dimensions are given in the table. The other parameters are dimensionless, as are the variables of the transformed maximization problem.

Table 1
List of Parameters

Parameter	Value	Explanation
γ	0.1	Relative discount rate
x_0	0.15	Open-access density
b_0	1.5	Relative demand parameter
p	10	Price parameter (NOK per kilogram)
K	6,000	Carrying capacity (1,000 kilograms)

[8] For example, the analytical expression for the equilibrium solution of one of the variables fills several pages. To interpret such expressions is beyond our scope.

The Premium of Marine Protected Areas

Figure 1 shows the result from numerical calculations of the premium of an MPA as a function of s for four different values of the normalized migration rate. There are three important things to observe in this figure: (*i*) the premium is positive and increasing with s; (*ii*) increasing the normalized migration rate reduces the premium; and (*iii*) the premium is smaller than s, which is the same as saying that you have to give up less in profit than what you protect in habitat, $P(s) < s$. The first observation (*i*) is a consequence of the fact that the model is deterministic and nonlinear and that we optimize returns in the unprotected area. The second observation (*ii*) gives us reason to comprehend the normalized migration rate as a degree of exploitation of the stock in the reserve. The higher the rates, the less profits decline and, as we will see in a moment, harvest is reduced by a smaller amount. The last observation (*iii*) results from several mechanisms, but the driving force behind all of them is the fact that density levels change. Changing densities influence profits directly through the *stock effect* and indirectly through the *market effect*; demand is elastic and prices change when harvests change. Harvests change due to changes in production. Changes in production relate directly to the changing stock densities. Migration, which in this context can be perceived as a part of the production system, also

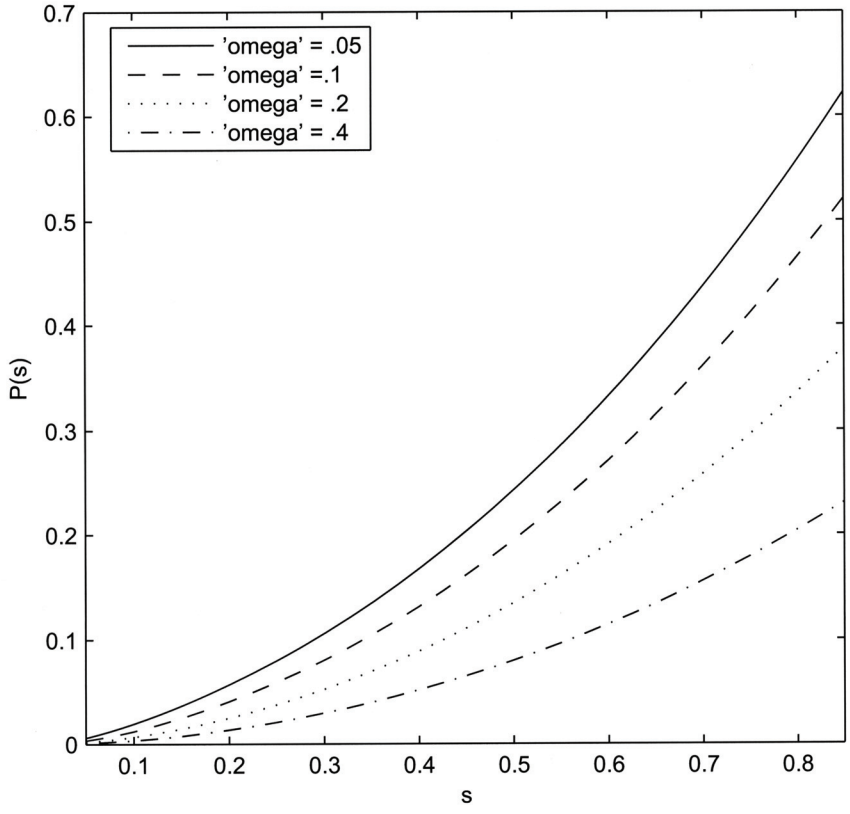

Figure 1. The MPA Premium as a Function of the Degree of Protection, $P(s)$
Notes: Parameters are given in table 1. The different curves correspond to different values for the normalized migration rate, ω.

changes due to changing densities. We shall investigate all of these mechanisms later, but before that, one more observation is necessary in figure 1.

Note that the premium curves are *convex*. Since the curves are increasing, convexity means that protecting one extra unit of habitat is more costly than the last unit. For example, by trying to make a protectionist twice as happy by doubling the size of the reserve, it more than doubles the difficulty for the commercial agents in the fishery. This presumably complicates, *e.g.*, the process of deciding on the degree of protection. However, convex costs are in agreement with fundamental economic assertions and usually make it easier to guarantee the existence and uniqueness of an internal solution in economic problems.[9] We observe convexity also with a zero migration rate. This is analytically feasible and is carried out in the appendix. The analytical results improve our confidence in the generality of the convexity property. Further, it suggests that the convex property stems from the nonlinearities in the profit function; *i.e.*, the *market effect* and the *stock effect*.

In figure 2(a), we compare the premium from the equilibrium solution in figure 1 with the premium arising from the dynamic solution for two different initial density levels (0.1 and 1). (Please note that we compare different situations: one where the density is uniform at time zero; that is, when the reserve is established; and one where the initial density distribution matches the equilibrium distribution. In the former situation, the system evolves and presumably converges towards the equilibrium; *i.e.*, a dynamic situation. The latter situation is static. Such 'static' initial conditions are by all practical means impossible. Still, the results indicate that the comparison is justifiable.) Notice that the equilibrium curve, which is based on the assumption that the equilibrium density is the initial density, is found between the curves of the dynamic solutions. This happens as the initial densities are lower (or higher) than the equilibrium density for all degrees of protection. That is, the larger the stock when the MPA is established, the cheaper it becomes. However, we also observe that the gap between the two dynamic solutions is rather small. This indicates that the initial size of the stock is not very important for the premium of an MPA. Remember that our definition of the premium is a relative measure and that the absolute premium related to highly profitable stock levels is much higher than that of small stock levels, which are less profitable. Still, our results suggest that using equilibrium solutions when calculating the MPA premium is a good approximation, and its simplicity outweighs the error of doing so. Taking into consideration that many of the input parameters are only roughly known, the importance of this simplification is further diminished. We will use this approximating approach for the rest of the article (that is, only considering steady-state values of the variables), except when dealing with time-series solutions. For the sake of completeness, the appendix provides an example of a feedback harvest rule and the corresponding value function for our problem (figure A1). However, the results in figure 2(a) depend on the rate of discounting. We use a rate of 5% in the calculations reported throughout the article. The equilibrium solution is representative for the dynamic solutions to a decreasing degree for an increasing rate of discounting. Still, after investigating the results for a range of discount rates (figures 2[b]-[d]), we feel confident that our equilibrium analysis gives a good estimation of how these are connected, at least for rates up to 20%.

We shall now investigate the mechanisms behind observation (*iii*) in figure 1, $P(s) < s$. As already stated, the driving force behind the different effects is the changing density levels as the degree of protection changes. Remember that we are considering the equilibrium solution, so what really happens is that there are differ-

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[9] We are indebted to an anonymous referee for this insight.

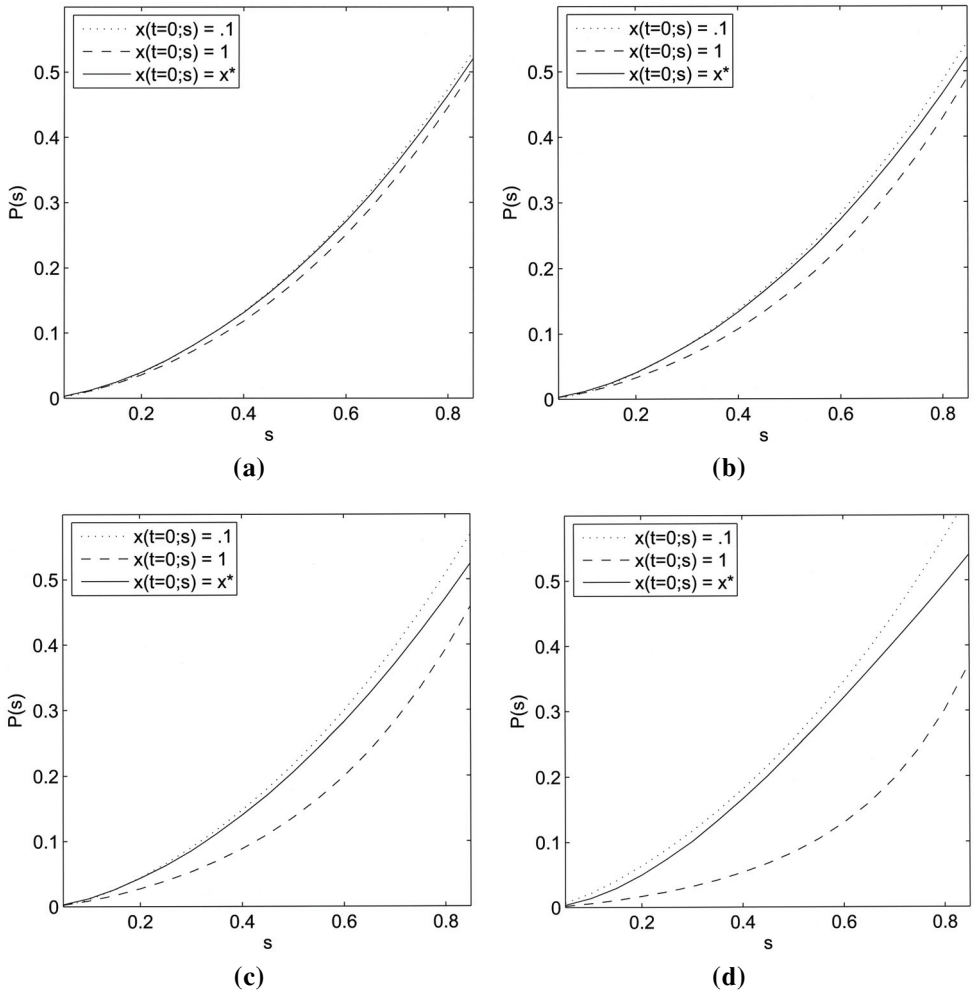


Figure 2. Comparing the Static (equilibrium, x^*) and Dynamic Solutions of the Premium for Two Different Initial Density Conditions: $x(t = 0; s) = 0.1$, and $x(t = 0; s) = 1$

Notes: The four panels correspond to four different rates of discounting: (a) 5%, (b) 10%, (c) 20%, and (d) 50%. These are all per annum rates. Other parameters are given in table 1. The normalized migration rate is held fixed at $\omega = 0.1$.

ent optimal equilibrium positions for different protection levels. Equilibrium density levels and the harvest rate as functions of the degree of protection are displayed in figure 3(a). We observe that the density in the protected area is increasing while decreasing in the rest of the habitat. The increase in density in the protected area reduces production in that area; it moves away from the level of MSY, which is exactly one half.[10] Production increases in the remaining grounds as long as the

[10] The absolute production may well be rising. As the area size is rising, the actual biomass in it increases faster when both the size and the density are rising. However, we are interested in the densities and how *efficient* production is. When the density level moves away from the MSY level, production is less efficient, or a smaller part of the potential for natural growth is realized.

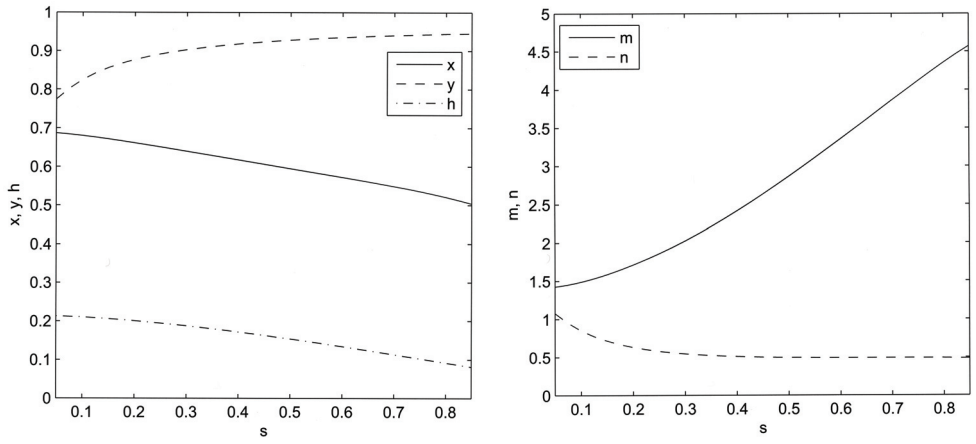


Figure 3. (a) Density Levels, x and y, and Harvest Rate, h, as Functions of the Degree of Protection; (b) Corresponding Shadow Values for the Stock Levels in (a); m Corresponds to x, n Corresponds to y

Notes: The normalized migration rate is $\omega = 0.1$. Other parameters are given in table 1.

density is above the MSY level, approaching it as the density is decreasing. As the density falls beyond the MSY level, production decreases also in the unprotected area. In absolute terms, the aggregated production is declining as s is increasing in the entire interval. The falling density in the unprotected area also implies that harvesting costs are increasing. The decline in harvest (figure 3[a]), which is in accordance with the findings of Guénette and Pitcher (1999), induces a higher market value of a unit of fish due to the demand mechanism.[11] It turns out that the optimal harvesting strategy is to put stronger pressure on the unprotected stock and reduce the density. The gain in the unit price of fish is larger than the increase in the unit harvesting cost. Thus, for a small agent in the fishery with an individual, fixed quota, the fishery becomes more profitable. Note that the property $y > x$ is satisfied.

We have identified three mechanisms that have influence on the value of the fishery: rising costs from declining density (*stock effect*), decline in harvest (which we coin the *protection effect*), and rising prices (*market effect*). The first two of these have negative impact on returns; the latter has a positive effect. As already established, the negative effects outweigh the positive effect with increasing strength along s and decreasing strength along ω. We will return to the interplay between the *stock effect* and the *market effect* when we turn to time-series solutions.

Related *shadow values* (refer to equation [8]) are found in figure 3(b). The shadow values are interpreted as the marginal value of the associated state variables. In the case of a fish stock, it measures the value of fish left unharvested, or rather, the value of an additional unit of fish in the sea. Not surprisingly, the value of that extra unit of unprotected fish increases as more of the habitat is protected. As the degree of protection increases, the supply of the resource is reduced. As already

[11] Note that the scaling of the harvest variable depends on the degree of protection. The scaling transformation is singular in $s \to 1$, however. We control for the dependence on s in the reported results, which then are easier to interpret.

seen, the density in the unprotected area is decreasing. The shadow value of the protected stock displays some interesting behavior. For small reserves the shadow value is falling, and for large reserves it is rising. The increase for large s is hardly visible in the figure. The effect is more pronounced for smaller levels of b; that is, a weaker *market effect*. An additional unit of protected fish has two consequences: production decreases in the protected area since the stock level moves further away from the MSY stock level, and migration increases since the difference in density increases. By studying the shadow price we increase our understanding of the changing densities, and thus the mechanisms behind the premium curves in figure 1. As observed in figure 3(a), the stock increases rapidly for small reserves and slower for large reserves. A possible explanation for the behavior of the shadow value for the protected stock may be that the negative impact on production is stronger than the positive impact on migration for small s, and vice versa for large s.

Biological Measures

To increase the understanding of the model dynamics, we construct and compute the *effect on relative growth* (*ERG*) for the two areas, which tells us how much of the potential production is reached in each area. This is a very common measure (sometimes even subject to maximization), particularly in purely biologically oriented papers (*e.g.*, Guénette and Pitcher 1999). We use MSY as a benchmark, hence we have $ERG(\cdot) = 1$ when MSY is attained. Note that we consider densities and consequently the growth function is $f(x) = x(1 - x)$. We have $x_{MSY} = 1/2 \Rightarrow f(x_{MSY}) = 1/4$. The absolute migration between the areas is also calculated. The results are found in figures 4(a) and (b). The effect on growth depends on the density level and is given by:

$$ERG(x) = \frac{f(x)}{f(x_{msy})} = 4 \cdot f(x),$$

where x is the density in the unprotected area. $ERG(y)$ is defined similarly for the reserve; the argument indicates which area is addressed. The absolute growth depends on the actual size of the areas. Observe that even though the density level in the protected area is moving away from the MSY level (figure 3); that is, growth is decreasing, migration is increasing for all s (figure 4[b]). The gap between the densities increases (figure 3). The production in the reserve must equal the migration, hence the absolute production is increasing. Nevertheless, it becomes less efficient. Moreover, the results in figure 4(b) suggest that the migration is increasing in the migration rate.[12] It is also interesting that the $ERG(x)$ is more or less unchanged for different migration rates, while $ERG(y)$ changes considerably. Looking further into this, figures 4(c) and 4(d) compare migration with the natural growth in the two areas. That is:

$$\frac{\phi(y - x)}{F(X; (1 - s)K)},$$

for the open area and correspondingly for the protected area. We find that the relationship responds heavily to changes in the migration rate when it comes to the open

[12] The analysis regards the normalized rate of migration. However, as the normalization is linear in the migration rate, the monotonicity results hold.

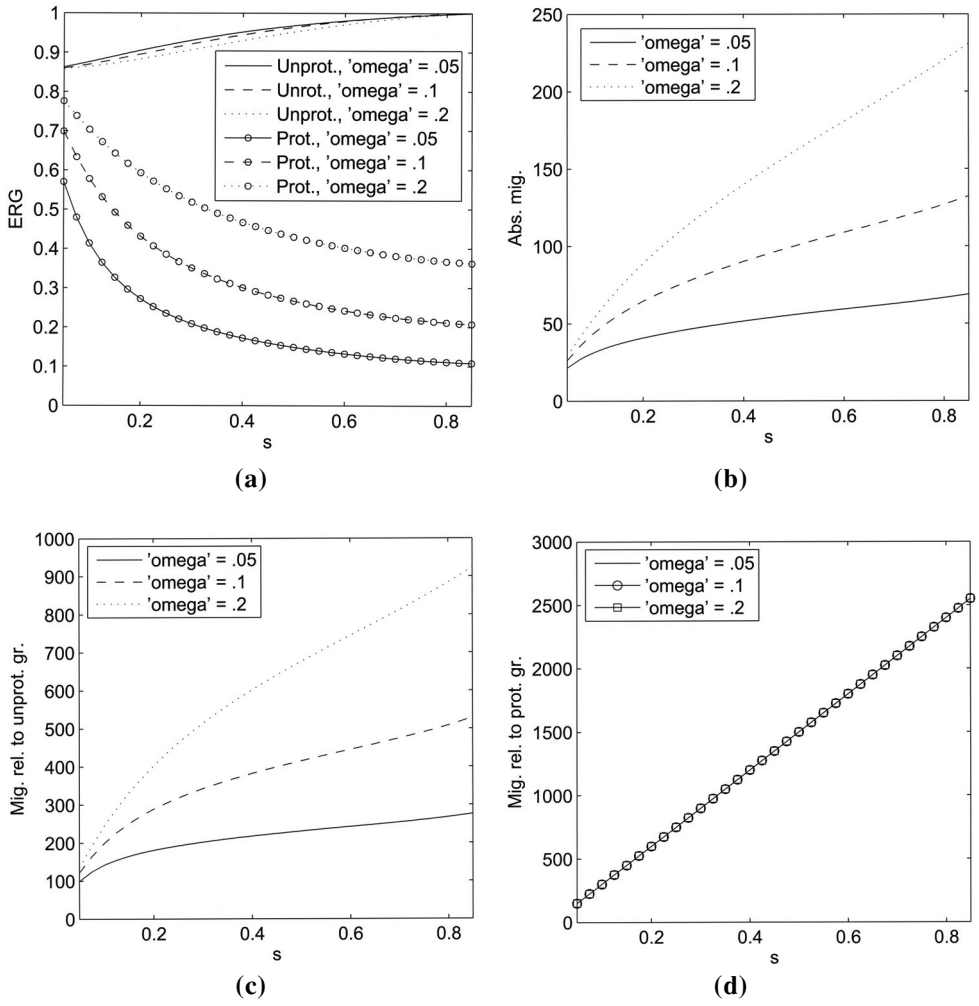


Figure 4. (a) The *Effect on Relative Growth* (*ERG*) in the Two Areas as Functions of *s*, Displayed for Three Different Normalized Rates of Migration; (b) Measuring the Absolute Migration between the Two Areas, given by $\phi(y - x)$; (c) Migration Relative to the Natural Growth in the Unprotected Area; (d) Migration Relative to the Natural Growth in the Protected Area

Note: Parameters for all four panels are found in table 1.

area (figure 4[c]); production remains more or less the same while the migration changes. As established from the dynamic constraints, there is a one-to-one relationship between migration and production in the protected area (figure 4[d]).

We study two more biological measures. The first is the relative *increase in standing stock* (*ISS*) from establishing an MPA. Hannesson (1998) coins this measure the *conservation effect*. The increase in stock is an increase in biological capital; a biological gain. Note that, particularly in terms of production, an increase in stock is not always positive. However, when there is no upper limit on harvesting

capacity, an increase in stock represents an increase in value of the fishery for any relevant stock-level. This is exemplified in figure A1(b). For the *ISS* measure, we need to consider the absolute values. We have $X(s) + Y(s) - X(0)$, where $X(s)$ and $Y(s)$ are the absolute standing stock levels for different degrees of protection, and $X(0)$ is the standing stock level for the no protection case. However, absolute terms are a bit awkward, and by dividing through by $X(0)$, we transform the measure into a relative measure. The relative *ISS* is given by:

$$ISS(s) = \frac{X(s) + Y(s)}{X(0)} - 1.$$

The measure compares the size of the equilibrium standing stock under the two different strategies: reserve and no reserve. It turns out that the shape of these curves is similar to the shape of the premium curves (figure 1). Thus, we find it natural to compare these; that is, in figure 5 we consider the relationship $ISS(s)/P(s)$. This relationship compares the relative increase in biomass (capital) with the relative loss in

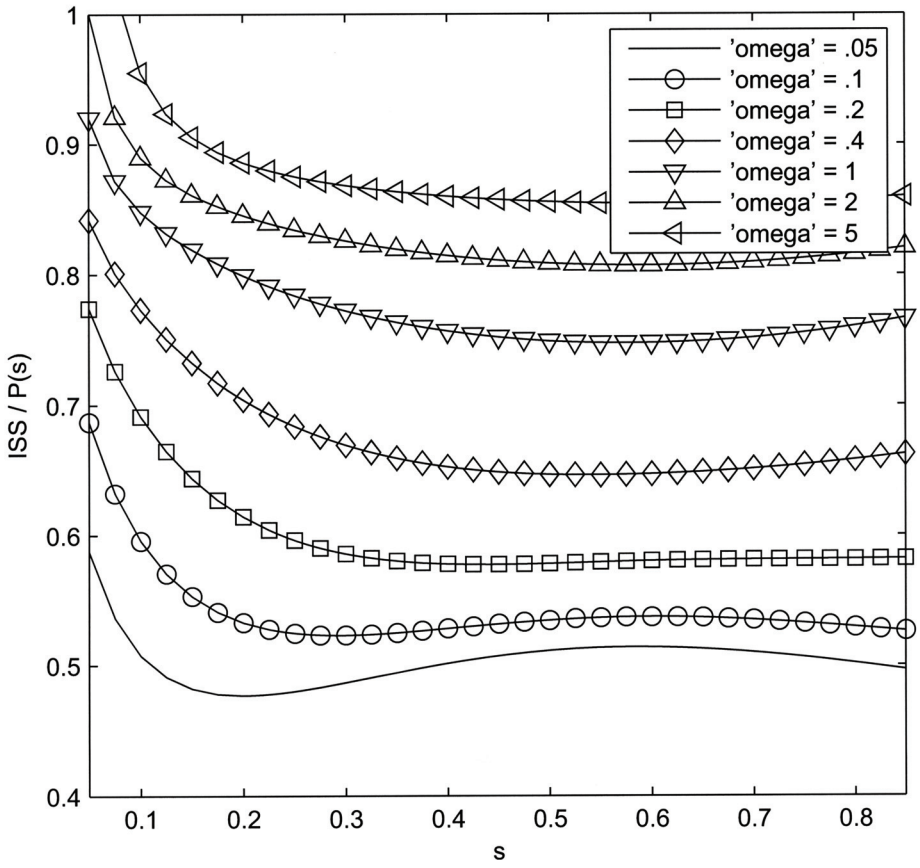


Figure 5. The Relative *Increase in Standing Stock, ISS*(*s*), Compared to the Premium of MPAs, $ISS(s)/P(s)$, for Different Values of the Normalized Migration Rate
Note: Parameters for the calculations are found in table 1.

total profit. Figure 5 shows the results for a range of different values for the normalized migration rate. In order to provide a complete analysis, some of the corresponding curves for $ISS(s)$ are presented in figure 6.

The evidence from figure 5 is that the premium dominates the increase in standing stock $\{[ISS(s)/P(s)] < 1\}$ for all degrees of protection and for most rates of migration. A higher normalized migration rate dilutes the *conservation effect*. This is an important mechanism behind the reduction in the premium from increasing the rate of migration.[13]

While $ISS(s)$ measures how much the MPA strategy increases the standing stock, the measure we construct next concerns the share of the standing stock that is protected. From the property $y > x$ we know that the density is higher in the reserve, thus the protected share of the total stock is larger than the protected share of habi-

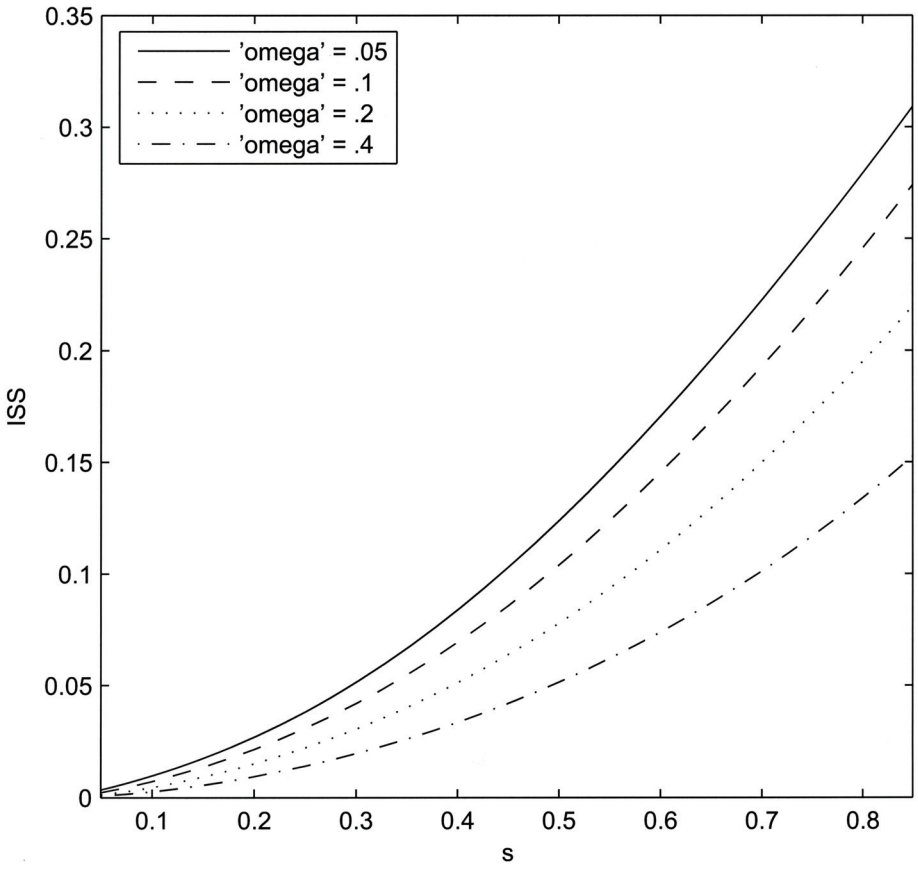


Figure 6. The Relative *Increase in Standing Stock, ISS*(*s*), for Four Different Values of the Normalized Migration Rate

Note: Further parameters are found in table 1.

[13] Moreover, the results imply that the premium declines faster than the increase in stock with the migration rate. The slope of the curves in figure 5 is determined by the relationship between the slopes of $ISS(s)$ and $P(s)$. For small s, the premium curve has the steeper slope and vice versa for large s.

tat, s. What we want to know is exactly how much more than s of the biomass is found under protection. At least s of the biomass is found under protection with certainty for any nonnegative migration rate. We can perceive this as a measure of how *effective* the MPA is to protect the stock. We name this measure the *protected standing stock* (*PSS*). That is:

$$PSS(s) = \frac{Y(s)}{X(s) + Y(s)} - s. \tag{10}$$

Figure 7 displays $PSS(s)$ for different values of the normalized migration rate. It must be zero in $s = 0$ and $s = 1$ and positive for all other $s \in (0,1)$, given that the rate of migration is positive and finite. This follows from the equilibrium property $y > x$. $PSS(s)$ decreases with the migration rate. This is in accordance with Guénette and Pitcher (1999), *inter alios*. This is also in accordance with our earlier results. A high rate of migration reduces the *conservation effect*, and as the migration rate goes to

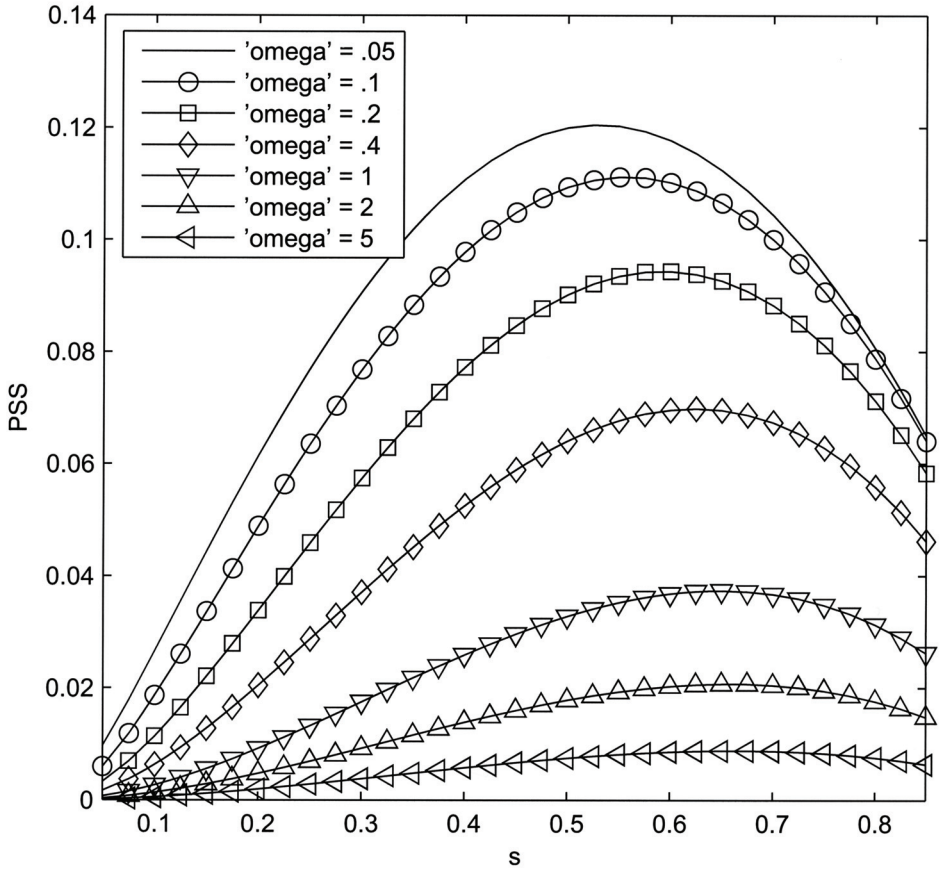


Figure 7. Protected Standing Stock, $PSS(s)$, for Different Values of the Normalized Migration Rate

Note: Other parameter values are found in table 1.

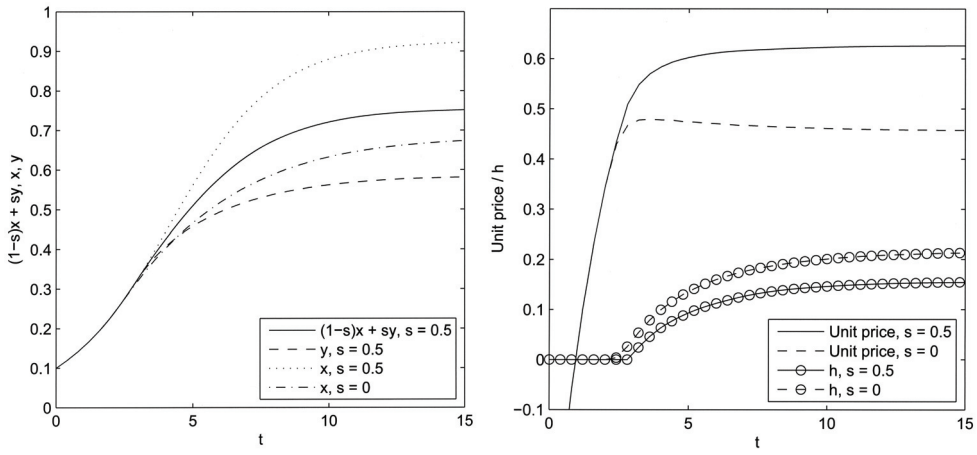
infinity, the density approaches a uniform distribution: $\lim_{\phi \to \infty} PSS(s) = 0$. The expression in (10) arises from comparing the natural growth, or production, when $s = 0$ and $s > 0$. The comparison suggests that whenever s equals the share of the stock found in the protected area; *i.e.*, that the density is uniform in the entire habitat ($x = y$), the growth in both cases is equal. This is straightforward as the growth is density dependent. Different values of s yield a smaller total growth. The foregone production is a consequence of the convexity of the logistic function and the dispersed densities (refer to the earlier discussion of difference in growth). $PSS(s)$ assess the *displacement* between the best possible dispersion when it comes to total, natural growth (uniform dispersion) and the equilibrium dispersion. Thus, we can comprehend $PSS(s)$ as a measure of *inefficiency* in the natural production. The analytical comparison of the two cases and the derivation of (10) are found in the appendix.

Time-Series Solutions

Until now, we have studied the equilibrium solution of the problem. We will finally turn to the dynamic solutions. We present a comparison of two time-series solutions: one with and one without an MPA. Or rather, we compare the solution of the corresponding one-dimensional model with the solution of the MPA model. We investigate the density distribution, harvest rate, and real unit price of fish as functions of time. The MPA covers half the habitat of the fish stock. Figure 8(a) shows density paths where the initial stock level is only 10% of the total carrying capacity. The fish are uniformly dispersed in the two areas at time zero; the densities in the two areas are equal. The figure also displays the average fish density in the MPA case, and the curve is thus comparable to the one-dimensional case. Corresponding catch rates are found in figure 8(b). We observe that the average density is always higher with the reserve; consequently, the total standing stock is larger in the MPA situation. Further, the density in the unprotected area is smaller than the density in the no reserve case; the *stock effect* induces higher harvesting costs with the reserve implemented. The catch rate is always higher without the reserve. Further, the system approaches equilibrium slower when there is no reserve. There is a 'moratorium' in the initial period until the stock has been rebuilt to levels above the open-access level. The 'moratorium' is lifted earlier in the no-MPA case. The two systems (MPA and no MPA) follow the same path as long as harvest is zero; that is, as long as the density is uniformly dispersed. In figure 8(b) the real unit price of fish is calculated. Notably, the price is always higher in the reserve case. We have two effects in play here: changes in harvest level change the price of fish in the market, and changing densities influence costs related to catching. A higher real price in the MPA case implies that the *market effect* outruns the *stock effect*, and the *net return* per unit of fish increases with the reserve in place. There is typically room for fewer active agents in the fishery, however, as the total catch is smaller. These results depends on the relationship between the price parameter, p, and the demand parameter, d. Certainly, parameters exist (although more obscure) such that the conclusion is reversed; a weaker *market effect* compared to the *stock effect*, and the fishery becomes less profitable for the agents as a consequence of the establishment of a marine reserve.

Conclusions

Hannesson (2002) is skeptical to advocate marine protected areas as a bet hedging tool, and Sanchirico (2000) dismisses the use of the term *insurance* related to reserves. However, there is a wide consensus that MPAs have the potential to reduce



(a) **(b)**

Figure 8. (a) Densities as Functions of Time, *t*. Two Protection Scenarios are Compared: $s = 0.5$ and $s = 0$. The Aggregate Density is given by $(1 - s)x + sy$ in the Reserve Scenario (solid line). Local Densities in the First Scenario are also Reported. (b) Comparing the Harvest Rate and the Real Unit Price of Fish in the Same Two Scenarios

Notes: The normalized migration rate is set to $\omega = 0.1$ in both panels, time is given in years, and further parameter values are found in table 1.

uncertainties in several aspects of a fishery. To be able to evaluate the usefulness of MPAs, thorough analyses of economic and biological effects are required, particularly when such instruments are implemented. Grafton, Kompass, and Schneider (2005, p. 173) name the "small number of studies that combine both the biological and economic drivers of marine reserves" as one of the main barriers in better utilization of reserves. In this article, we have gone to new depths in analyzing the cost incurred from establishing a protected area in a bioeconomic model. Particularly, the implementation of a two-dimensional optimal harvest rule is a new contribution. We reveal and discuss how costs are generated and study the different mechanisms that work on both economic and biological factors. Different biological measures are proposed and investigated.

The main finding is that costs are smaller than the share of protected habitat compared to the total profits, and that protecting one more unit is more costly than the last unit protected; *i.e.*, convexity. Convex costs agree with standard economic assertions and increase confidence in the results. However, the lack of knowledge on the migration term is a problem. The optimization indicates that the harvest rate should increase in the remaining area open to fishing as the degree of protection increases. That is, the larger the protected are is, the more one wants to stimulate migration through a large density gap. The results are based on numerical calculations, and the generality depends on the choice of parameter values. However, the model displays mostly monotonic behavior when we alter the different parameters for large intervals around the investigated parameters. This leads us to believe that our findings apply more generally. In particular, the convex property is well documented, as it is shown analytically for the zero migration case.

We have investigated different biological measures. We find that a reserve leads

to less efficient biological production. Results suggest that the rate of migration is an important determining factor when comparing the relative increase in the standing stock with the premium of the MPA. Further, the share of stock found in the reserve compared to that of protected habitat is investigated. This measure could be understood as the marginal biological value of the protection; it demonstrates the increase in protected biomass relative to the degree of protection.

Our analysis also shows that the fishery potentially becomes more profitable to individual agents (remaining active) because the real unit price of fish goes up. However, this effect is critically dependent on the chosen parameters.

Finally, we think there is room for improvement and further investigation within this framework. In particular, we would like to see the results from introducing a stochastic element in the model. Studying more sophisticated biological and economic submodels will potentially provide additional insights.

References

Anderson, L.G. 2002. A Bioeconomic Analysis of Marine Reserves. *Natural Resource Modeling* 15(3):311–34.

Apostolaki, P., E.J. Milner-Gulland, M.K. McAllister, and G. Kirkwood. 2002. Modelling the Effects of Establishing a Marine Reserve for Mobile Fish Species. *Canadian Journal of Fisheries and Aquatic Sciences* 59:405–15.

Arnason, R. 2001. Marine Reserves: Is there an Economic Justification? *Fisheries Centre Research Reports*, vol. 9 (8), R. Sumaila, ed., pp. 19–31. British Columbia, Canada: Fisheries Centre, University of British Columbia.

Bertsekas, D.P. 2005. *Dynamic Programming and Optimal Control*, vols. 1-2, 3rd ed. Belmont, MA: Athena Scientific.

Bischi, G.I., F. Lamantia, and L. Sbragia. 2006. Strategic Interaction and Imitation Dynamic in Patch Differentiated Exploitation of Fisheries (submitted). Available at www.econ.uniurb.it/bischi/bischi_publications.html

Clark, C.W. 1990. *Mathematical Bioeconomics: The Optimal Management of Renewable Resources*, 2nd ed. USA: John Wiley & Sons, Inc.

___. 1996. Marine Reserves and the Precautionary Management of Fisheries. *Ecological Applications* 6(2):369–70.

Conrad, J.M. 1999. The Bioeconomics of Marine Sanctuaries. *Journal of Bioeconomics* 1:205–17.

Flaaten, O., and E. Mjølhus. 2005. Using Reserves to Protect Fish and Wildlife—Simplified Modeling Approaches. *Natural Resource Modeling* 18(2):157–82.

Fournier, D.A., J. Hampton, and J.R. Sibert. 1998. Multifan-cl: A Length-based Age-structured Model for Fisheries Stock Assessment, with Application to South Pacific Albacore, *Thunnus alalunga*. *Canadian Journal of Fisheries and Aquatic Sciences* 55:2105–16.

Gell, F.R., and C.M. Roberts. 2003. Benefits Beyond Boundaries: The Fishery Effects of Marine Reserves. *TRENDS in Ecology and Evolution* 18(9):448–55.

Grafton, R.Q., T. Kompass, and V. Schneider. 2005. The Bioeconomics of Marine Reserves: A Selected Review with Policy Implications. *Journal of Bioeconomics* 7:161–78.

Grüne, L., and W. Semmler. 2004. Using Dynamic Programming with Adaptive Grid Scheme for Optimal Control Problems in Economics. *Journal of Economic Dynamics and Control* 28:2427–56.

Guénette, S., and T.J. Pitcher. 1999. An Age-structured Model Showing the Benefits of Marine Reserves in Controlling Overexploitation. *Fisheries Research* 39:295–303.

Halpern, B.S. 2003. The Impact of Marine Reserves: Do Reserves Work and does Reserve Size Matter? *Ecological Applications* 13(1):117–37.

Hannesson, R. 1998. Marine Reserves: What Would they Accomplish? *Marine Resource Economics* 13:159–70.

___. 2002. The Economics of Marine Reserves. *Natural Resource Modeling* 15(3):273-90.

Hastings, A., and L.W. Botsford. 1999. Equivalence in Yield from Marine Reserves and Traditional Fisheries Management. *Science* 284:1537–38.

Holland, D., and R. Brazee. 1996. Marine Reserves for Fisheries Management. *Marine Resource Economics* 11:157–71.

Kamien, M.I., and N. Schwartz. 1991. *Dynamic Optimization: The Calculus of Variations and Optimal Control in Economics and Management*, 2nd ed. *Advanced Textbooks in Economics 31*, C.J. Bliss and M.D. Intriligator, eds. The Netherlands: Elsevier B.V.

Kugarajh, K., L.K. Sandal, and G. Berge. 2006. Implementing a Stochastic Bio-economic Model for the North-east Arctic Cod Fishery. *Journal of Bioeconomics* 8(1):35–53.

Lauck, T. 1996. Fisheries and Uncertainty, A Precautionary Approach to Resource Management. *Uncertainty in Fisheries Management*, D.V. Gordon, and G.R. Munro, eds., pp. 91–105. Calgary, Alberta, Canada: University of Calgary Press.

Lauck, T., C.W. Clark, M. Mangel, and G.R. Munro. 1998. Implementing the Precautionary Principle in Fisheries Management through Marine Reserves. *Ecological Applications* 8(1):72–8. Supplement.

McGarvey, R. 2004. Estimating the Emigration Rate from Marine Sanctuaries using Tag-Recovery Data. *Fishery Bulletin* 102(3):464–72.

McGarvey, R., and E. Feenstra. 2002. Estimating Rates of Fish Movement from Tag Recoveries: Conditioning by Recapture. *Canadian Journal of Fisheries and Aquatic Sciences* 59:1054–64.

Quinn, T.J., and R.D. Deriso. 1999. *Quantative Fish Dynamics*. New York, NY: Oxford University Press, Inc.

Sanchirico, J.N. 2000. Marine Protected Areas as Fishery Policy: A Discussion of Potential Costs and Benefits. *Resources for the Future Discussion Papers* 00-23, Washington, DC.

Sanchirico, J.N., and J.E. Wilen. 1998. Marine Reserves: Is there a Free Lunch? *Resources for the Future Discussion Papers* 99-09.

___. 2002. The Impacts of Marine Reserves on Limited-entry Fisheries. *Natural Resource Modeling* 15(3):291–310.

Sandal, L.K., and S.I. Steinshamn. 1997. A Feedback Model for the Optimal Management of Renewable Natural Capital Stocks. *Canadian Journal of Fisheries and Aquatic Sciences* 54:2475–82.

___. 2001. A Simplified Approach to Optimal Resource Management. *Natural Resource Modeling* 14(3):419–32.

Sumaila, U.R. 2001. Marine Protected Area Perfomance in a Game Theoretic Model of the Fishery. *Fisheries Centre Research Reports*, vol. 9(8), R. Sumaila, ed., pp. 229–235. British Columbia, Canada: Fisheries Centre, University of British Columbia.

Appendix

Comparing Growth Models

In absolute terms, the total natural growth in the entire area in the MPA model less the total natural growth in the corresponding one dimensional model is given by:

$$F(X; (1 - s)K) + F(Y; sK) - F(X + Y; K)$$
$$= rX\left(1 - \frac{X}{(1 - s)K}\right) + rY\left(1 - \frac{Y}{sK}\right) - r(X + Y)\left(1 - \frac{X + Y}{K}\right).$$

This expression should be equal to zero if there is no difference in the two growth models. The first factor in each term and the carrying capacity cancel each other, thus:

$$(X + Y)^2 - \frac{X^2}{1 - s} - \frac{Y^2}{s} = 0. \tag{A-1}$$

Solving this equation for s yields the double root:

$$s_0 = \frac{Y}{X + Y},$$

which shows that the growth is equal in the two models when the fraction of biomass in the two areas is proportional to the fraction of habitat under protection. Whenever that is the case, the density is uniform in the entire habitat; *i.e.*, the situation is identical in the two different models. Equal growth would then be anticipated. The expression for s_0 appears in (10).

Further, s_0 being a double root implies that the expression in (A-1) is either positive or negative for all $s \neq s_0$. It is readily seen that the expression is negative. Hence:

$$F(X; (1 - s)K) + F(Y; sK) \leq F(X + Y; K).$$

The natural growth in the corresponding one-dimensional model is always bigger or equal to the total natural growth in the MPA model.

Details on the Numerical Solution Scheme

The dynamic programming operator, T, is given by:

$$T_{\Delta\tau}[v_{\Delta\tau}](\mathbf{x}) = \max_{h \in [0,\infty)} \left[\Delta\tau \cdot \pi(\mathbf{x}, h) + \beta \frac{\partial v_{\Delta\tau}(\mathbf{x})}{\partial \mathbf{x}}\right], \tag{A-2}$$

where $\mathbf{x}$ is the discrete state variable; h is the control variable; $\Delta\tau$ is the discrete time step; $v_{\Delta\tau}(\mathbf{x})$ is the value function with respect to the time step and discrete state variable; and $\beta = 1 - \Delta\tau \cdot \gamma$, the discrete discount rate. The discrete variables, time

step, and discount rate are formulated in accordance with the discretization procedure given in Grüne and Semmler (2004). Note that we have omitted the s-parameter in the value function. To maximize our problem (5, 6), we approximate a solution to the fixed point equation:

$$v_{\Delta\tau}(\mathbf{x}) = T_{\Delta\tau}[v_{\Delta\tau}](\mathbf{x}).$$

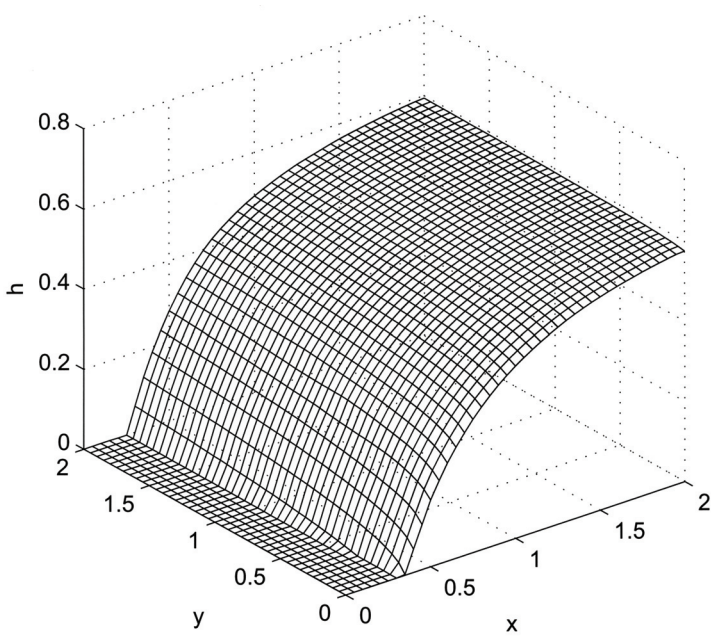
Solutions are produced from an iterative process (Bertsekas 2005; Grüne and Semmler 2004). The numerical solutions are approximations, as we end the iterative process when an error measure satisfies a predetermined condition.

Convexity of Premium in the Zero Migration Rate Case

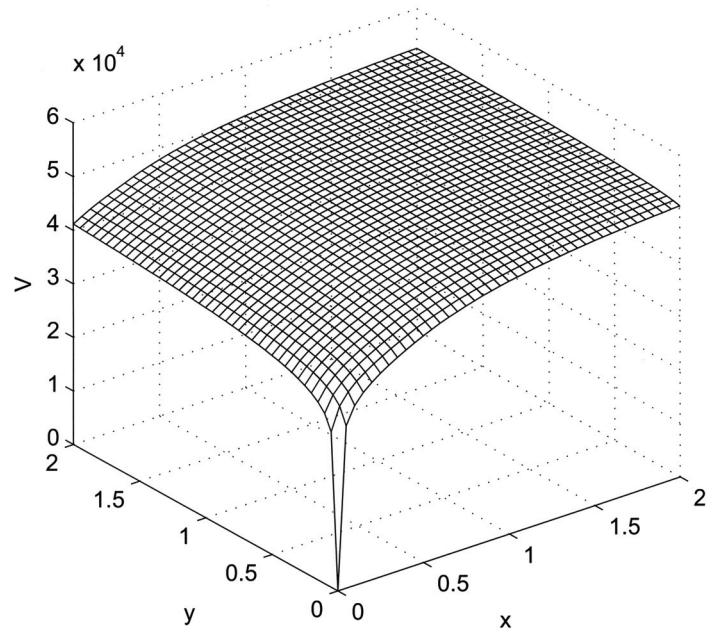
From equation (1) we see that the premium being convex in s is equivalent to $V(X^0, Y^0; s)$ being concave in s. When the migration rate is set to zero, the system degenerates to a one-dimensional system where the carrying capacity is reduced along s. However, as the profit function is concave in all variables, and all meaningful variables in the degenerate problem [*i.e.*, $X(s)$, $H(s)$] are decreasing monotonically in s, it is easily seen that $V(\cdot;s) = \Pi(\cdot)/\delta$ implies concavity in $V(\cdot;s)$ and thus convexity in $P(\cdot;s)$.

Additional Figures

Figure A1(a) displays an example of an optimal feedback harvesting rule. The corresponding value function is presented in figure A1(b). The value function yields the value of the fishery for given initial densities.



(a)



(b)

Figure A1. (a) Optimal Feedback Harvesting Rule as a Function of Local Densities; (b) the Value Function Corresponding to the Given Harvest Rule

Notes: The normalized migration rate is set to $\omega = 0.1$ in both panels, and $s = 0.5$. Further parameter values are found in table 1.